Further Inquiry into the Stabilities of Standardized and Structure Coefficients in Canonical and Discriminant Analyses

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Abstract

The stabilities of standardized (β) and structure (r_s) coefficients in canonical (CA) and discriminant analyses (DA) were studied. Four different situations were studied -- two pertaining to CA and two to DA. The situations were meant to represent "somewhat typical" and yet varying research conditions that often would not be thought to be notably objectionable among informed users of CA and DA.

Data were sampled from a real population. For each of three situations, 100 random samplings of size 100 each were performed. In each sampling β and r_s were computed, and were subsequently treated so that their stabilities could be evaluated. For one of the situations, 100 random samplings of size 150 each were performed.

Relative to the situations studied, conflicting results occurred concerning the stabilities of the β and r_s values. Conflicting results also occurred concerning the difference in stability between Roots 1 and 2 for both statistics. Furthermore, and more alarming, the stabilities of both statistics under the "reasonable" conditions studied were low. The results were discussed.

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Perspective and Point of View

Much evidence exists relative to the current interest in canonical and discriminant analyses concerning their relevance to data analysis pertaining to research in a variety of areas. A sampling of the evidence is as follows:

1. Numerous articles and books published and papers presented during the past several years relative to these multivariate methods. The scholarly works regarding the methods may be

found in social sciences, education, medical sciences, business, physical sciences, engineering, and other areas. Several works of classical significance also exist.

2. Papers produced by the following writers that are especially relevant to this paper: Heidgerken (1999); Roberts (1999); Strand (1999); Humphries-Wadsworth (1998); Thompson (1984, 1991, 1993, 1995a, 1995b, 1998); Kier (1997); Pedhazur (1997); Whitaker (1997); Gray, Baek, Woodward, Miller, and Fisk (1996); Jonathan, McCarthy, and Roberts (1996); Thomas and Zumbo (1996); Van de Geer (1996); Bewley and Yang (1995); Fan and Wang (1995); Fok and Fok (1995); Liang, Krus, and Webb (1995); Millns, Woodward, and Bolton Smith (1995); OGorman and Woolson (1995); Seo, Kanda, and Fujikoshi (1995); Tritchler (1995); Watts (1995); Yokoyama (1995); Beasley and Sheehan (1994); Cole, Maxwell, Arvey, and Salas (1994); Crossman (1994); Huberty (1975, 1994); Kingman and Zion (1994); Sadek and Huberty (1994); Hutchinson (1993); Kaplan and Wenger (1993); Kirisci and Hsu (1993); Mueller and Cozad (1993); Campbell and Tucker (1992); Chant and Dalgleish (1992); Harris (1989, 1992); Huberty and Wisenbaker (1992); Romanazzi (1992); Strand, Cahill, and Dirks (1992); Taylor (1992); Thomas (1992); Friedrich (1991); Joachimsthaler and Stam (1990); Thorndike and Weiss (1973); and Barcikowski and Stevens (1975).

Multivariate analysis of variance (MANOVA), discriminant analysis (DA), and canonical analysis (CA) are multivariate statistical techniques that are related to one another. MANOVA generally pertains to the relationship between one or more categorical independent (X) variables, and multiple continuous dependent (Y) variables.

DA generally pertains to the relationship between a single categorical Y variable and multiple continuous X variables. However, many users feel comfortable about the use of categorical X variables as long as they are coded appropriately. Furthermore, one may look upon a discriminant analysis as a "flip side" of a one-way MANOVA -- the single categorical X variable in a one-way MANOVA may be the single categorical Y variable in the corresponding DA, and the multiple Y variables in the one-way MANOVA may be the multiple X variables in the corresponding DA.

CA was initially developed in part to determine the relationship between a set of multiple continuous X variables and a set of multiple continuous Y variables. As practitioners became more knowledgeable about CA, some practitioners have become comfortable relative to the use of categorical variables as long as they are coded appropriately. Furthermore, other statistical techniques such as ANOVA, some MANOVAs and DAs, multiple regression analysis, and correlation may be looked upon as special cases of CA.

Wilks' lambda (Λ) pertains to the relationship between the X and Y variable composites taking into account all the solutions or roots.

Standardized coefficients, β, pertain to the relationship between a variable in one set and a variable in or variate (variable composite) of the other set controlling for the other variables in its own set. Structure coefficients, r_s, pertain to the relationship between a variable and the variate of its own set. These statistics are produced for each solution relative to a canonical or discriminant analysis. Furthermore, these statistics are frequently utilized in practice, and reports relative to their stability (degree to which their values may be cross validated across samplings) exist. However, the information that exists reflects conflicting points of view among notable statisticians -- for example, Tardif and Hardy (1995), Thompson (1984, 1991), Huberty (1975), and Barcikowski and Stevens (1975). While Thompson supported a point of view that structure coefficients are generally not necessarily more stable than standardized coefficients (Thompson referred to these as function coefficients), most theorists have argued and provided evidence that they are indeed more stable than the β values. Furthermore, the difference in the stability of these statistics across different solutions has received little attention.

That the value of standardized coefficients is questioned in multivariate analysis is not surprising in that several statisticians have looked upon them in a negative manner even in the use of relatively simpler multiple linear regression analysis [see Darlington (1990), for example].

Some of the previous studies were Monte Carlo studies relative to which samples were computer generated. Real data sets were utilized in other sampling studies.

A variety of approaches have been taken in evaluating the relative stabilities of the r_s and β coefficients (Strand, 1999; Fan & Wang, 1995; Tardif & Hardy, 1995; Thompson, 1991; Thorndike & Weiss, 1973; Barcikowski and Stevens, 1975; and Huberty, 1975).

The first writer's inquiry began several years ago -- resulting in related papers being presented at the annual meeting of the American Educational Research Association in 1998 and 1999 as well as other meetings. The inquiry initially focused on the relative stabilities of standardized and structure coefficients, and later included comparisons with the predictably more stable Wilks' lambda. Additional inquiry was made into the relative stability of the two statistics across multiple solutions.

These pursuits led to observations of the consistent and alarming lack of stability of both statistics even when the conditions that are required for unambiguous interpretation of the statistics appeared to be minimally violated. In the last set of studies, Strand (1999) selected variables for study (a) whose absolute skewness values generally fell below 1.00, (b) that resulted in relatively low collinearity, and (c) that were at least moderately related to variables in the other variable set. However, one of the 18 variables that were studied had a skewness value of 2.53 and kurtosis value of 18.53. Another variable had respective skewness and kurtosis values of -1.57 and 16.17. Relative to collinearity, three of the within-set *r* values exceeded .60 -- the greater of the two being .73. Furthermore, for the DA conditions studied, the relationship between the grouping variable and the linear combination of the discriminating variables was not high -- the respective population Λ values were .91 and .98. The results of these delimited studies added somewhat to the argument that structure coefficients are more stable than the corresponding standardized coefficients as well as providing consistent and alarming evidence of the relative instability of both statistics.

In this current and again descriptive study the writers have attempted to select even more appropriate variables for study -- variables whose skewness values and collinearity are even lower than in the previous studies, and variables that are even more related to variables in the other set of variables.

Hypotheses

The hypotheses relative to this paper are as follows:

1. Relative to each of CA and DA, the stability of r_s is greater than that of a corresponding β.

2. The stabilities of the β and r_s coefficients decrease in each subsequent solution or root.

3. The stabilities of both statistics are suitable under the conditions of low skewness and low collinearity, and relatively high relationship between the two sets of variables.

Method

Population

One population was utilized in this study. The population contained 1517 cases. One hundred samplings of size 100 or 150 (for the second set of DA runs only) were each performed for each of the following (the total sample sizes of 100 and 150 were selected so that the usable sample for each run after cases were eliminated because of missing data was at least 50):

1. DA runs concerning the first set of variables (NC, NS, A, HS, OPR, and CEDUC) relative to which Λ, β, r_s, and other values were produced.

2. CA runs concerning the first set of variables (HA, NC, NS, L, and E) relative to which Λ, β, r_s, and other values were produced.

3. DA runs concerning the second set of variables (OPR, OB, and COCAT) relative to which Λ, β, r_s, and other values were produced.

4. CA runs concerning the second set of variables (OCCA, NS, M, OPR, and PA) relative to which Λ, β, r_s, and other values were produced.

The first writer's experiences with previous studies showed that the results based on sample sizes of 50 and 100 were similar. Accordingly, sample sizes of 50 or more would likely produce results that are similar to what would be produced with moderately larger sample sizes.

The variables were selected in order to represent "somewhat typical" research conditions relative to which the researcher exercised appropriate caution in selecting the variables as follows:

1. An attempt was made to select all the continuous variables according to a criterion that their skewness values fell between -1.00 and 1.00, and all their kurtosis values fell below 4.00. Informed users of CA and DA are sensitive to the requirements for valid statistics and tests, and attempt to avoid variables whose distributions are markedly skewed. Table 1 contains skewness and kurtosis values for all the continuous variables selected for use. Most, but not all, of the variables met the criteria. NC and NS surpassed the upper criterion for skewness by a relatively small amount.

2. All the variables selected for the CA runs and all the discriminating variables selected for the DA runs were continuous variables.

Table 1

Skewness and Kurtosis Values for Continuous Variables

	DA runs			CA runs	
Variable	Skewness	Kurtosis	Variable	Skewness	Kurtosis
		Variable Set 1			
NC	1.03	1.06	HA	0.16	-0.53
NS	1.47	3.51	NC	1.03	1.06
A	0.52	-0.79	NS	1.47	3.51
HS	-0.24	1.10	L	0.29	-0.79
OPR	0.44	-0.37	E	-0.17	-0.71
		Variable Set 2			
OPR	0.44	-0.37	OCCA	0.65	-1.03
OB	-0.38	-1.20	NS	1.47	3.51
			M	-0.65	0.96
			OPR	0.44	-0.37
			PA	-0.18	-0.09

3. The number of cases falling in the groups of the two grouping variables utilized for the DA runs were not seriously unequal. Relative to the first set of DA runs, the number of cases falling in the five respective population groups of the grouping variable CEDUC after cases were eliminated because of missing data were 45, 86, 255, 285, and 82. Relative to the second set of DA runs, the number of cases falling in the two respective population groups of the grouping variable COCAT after cases were eliminated because of missing data were 244 and 673.

4. The variables in each of the X and Y sets of variables relative to the CA runs and the discriminating variables in the DA runs were selected to be somewhat but not highly correlated with each other. Informed users of CA and DA are sensitive to the variety of specification errors that may be committed when utilizing the techniques -- including within-set variables whose relationships are "too high." Tables 2 through 4 contain Pearson *r* values relative to the within-set variables selected for three of the four DA and CA runs. The within-set *r* values ranged from -.30 to .37. With regard to the second set of DA runs, the r value concerning the linear relationship between OPR and OB was .20.

5. The variables in each of the X and Y sets of variables relative to the CA runs were also selected to be somewhat correlated with the variables in the other variable set (see Tables 3 and 4). The between-set *r* values ranged from -.56 to .67.

A CA run was performed utilizing the first set of variables in the population. Selected “statistics” (in quotes because they pertain to a population) that pertained to this run were as follows:

1. $\Lambda = .76$, $F(6,1916) = 47.02$, $p = .00$.

Table 2

Pearson *r* Values Relative to Variables Selected from Variable Set 1 for DA Runs

			Variable		
Variable	NC	NS	A	HS	OPR
NC		.19	.37	-.22	-.09
NS			.12	-.22	-.16
A				-.23	.01
HS					.36

Table 3

Pearson *r* Values Relative to Variables Selected from Variable Set 1 for CA Runs

		X set Variable			Y set Variable	
Variable		HA	NC	NS	L	E
X	HA		.01	.03		
Set	NC			.19		
Y	L	.36	.07	.07		-.25
Set	E	-12	-.27	-.26		

Table 4

Pearson *r* Values Relative to Variables Selected from Variable Set 2 for CA Runs

		X set			Y set	
		Variable			Variable	
Variable		OCCA	NS	M	OPR	PA
X	OCCA		.18	-.21		
Set	NS			-.30		
Y	OPR	-.56	-.16	.15		.16
Set	PA	-.20	-.28	.67		

2. Solution 1 (Root 1) β values: HA = 0.71; NC = 0.44; NS = 0.46; L = 0.61; and E = -0.66. Root 2 β values: HA = 0.71; NC = -0.40; NS = -0.54; L = 0.84; and E = 0.80.

3. Root 1 r_s values: HA = .72; NC = .53; NS = .56; L = .77; and E = -.81. Root 2 r_s values: HA = .69; NC = -.49; NS = -.59; L = .64; and E = .59.

A CA run was performed utilizing the second set of variables in the population. Selected statistics that pertained to this run were as follows:

1. Λ =.38, $F(6, 1836) = 188.19$, $p = .00$.

2. Root 1 β values: OCCA = -0.32; NS = -0.14; M = 0.84; OPR = 0.32; and PA = 0.90. Root 2 β values: OCCA = 0.97; NS = -0.04; M = 0.51; OPR = -0.96; and PA = 0.46.

3. Root 1 r_s values: OCCA = -.51; NS = -.39; M = .94; OPR = .46; and PA = .95. Root 2 r_s values: OCCA = .86; NS = -.05; M = .31; OPR = -.89; and PA = .31.

Relative to the DA runs, the variables in the X set were selected to be somewhat related to the categorical grouping variable.

A DA run was performed utilizing the first set of variables in the population. Selected statistics that pertained to this run were as follows:

1. $\Lambda = .48$, $X^2\,(20) = 547.31$, $p = .00$.

2. Root 1 β values: NC = -0.06; NS = -0.18; A = -0.32; HS = 0.62; and OPR = 0.61. Root 2 β values: NC = 0.12; NS = 0.18; A = 0.77; HS = -0.01; and OPR = 0.49. Statistics for Roots 3 and 4 are not provided.

3. Root 1 r_s values: NC = -.24; NS = -.26; A = -.31; HS = .73; and OPR = .64. Root 2 r_s values: NC = .33; NS = .09; A = .86; HS = -.07; and OPR = .57. Statistics for Roots 3 and 4 are not provided.

A DA run was performed utilizing the second set of variables in the population. Selected statistics that pertained to this run were as follows:

1. $\Lambda = .50$, $X^2(2) = 640.14$, $p = .00$.

2. β values: OPR = 0.99; and OB = 0.07.

3. r_s values: OPR = 1.00; and OB = .18.

SDs were computed for the 100 β and r_s values obtained in the DA and CA runs for each of the following:

1. Each variable in the X set relative to each of the two DA sets of runs.

2. Each variable in both the X and Y sets relative to each of the two CA sets of runs.

The SDs of β and r_s for each variable were compared. While similar and other approaches were taken in previous studies, the writers acknowledge some limitations in comparing the SDs of the β and r_s distributions since the r_s values can only range from -1.00 to 1.00 while the absolute values for β may exceed 1.00. Ranges and interquartile ranges were also computed and compared.

SPSS 10.0 for Windows was used for the sampling and computations.

Results

DA Runs

Table 5 contains the results for the first set of DA runs. Relative to Solution 1, the SD for β was greater than the SD for r_s for three of the paired values. For Solution 2, the SDs for all the β values were greater than the SDs for all the corresponding r_s values. The ranges and interquartile ranges somewhat cross validated the SD results. Furthermore, in nine of ten cases the SD concerning Solution 2 was greater than the corresponding SD for Solution 1.

Table 6 contains the results for the second set of DA runs. While low, the SD for both the β values were greater than the SDs for the corresponding r_s values. The ranges and interquartile ranges generally cross validated the SD results.

CA Runs

Tables 7 and 8 contain the results for both the CA sets of runs. Relative to the first set of CA runs (Table 7), Root 1, the SD for β was lower than the corresponding SDs for all the five r_s values. Relative to Root 2, the SD for β was lower than the corresponding SDs for two of the five r_s values. The ranges and interquartile ranges somewhat cross validated the SD results. Furthermore, in seven of ten cases the SD concerning Solution 2 was less than the corresponding SD for Solution 1.

Relative to the second set of CA runs (Table 8), Root 1, the SD for β was lower than the corresponding SDs for all five r_s values. Relative to Root 2 the SD for β was lower than the corresponding SDs for one of the five r_s values. The ranges and interquartile ranges somewhat cross validated the SD results. Furthermore, in six of ten cases the SD concerning Solution 2 was less than the corresponding SD for Solution 1.

Table 5

Standard Deviation and Other Statistics Relative to Standardized and Structure Coefficients Concerning Variable Set 1 for DA Runs

	β			r_s		
Variable	Range	SD	Skewness	Range	SD	Skewness
			Solution 1			
NC	-0.51 to 0.50	.21	0.25	-.54 to .63	.19	1.35
	(-0.21 to 0.09)			(-.32 to -.12)		
NS	-0.53 to 0.62	.24	1.20	-.63 to .59	.22	1.77
	(-0.26 to -0.01)			(-.34 to -.18)		
A	-0.78 to 0.79	.28	1.16	-.55 to .50	.22	1.64
	(-0.44 to -0.09)			(-.38 to -.14)		
HS	-0.63 to 0.96	.28	-2.38	-.74 to .98	.31	-3.03
	(0.48 to 0.69)			(.56 to .76)		
OPR	-0.73 to 0.92	.32	-2.55	-.69 to .96	.32	-2.42
	(0.50 to 0.71)			(.49 to .69)		
			Solution 2			
NC	-0.97 to 0.97	.50	-0.17	-.59 to .91	.39	-0.31
NS	-0.64 to 0.83	.30	-0.29	-.54 to .75	.27	-0.09
A	-0.80 to 1.05	.45	-1.18	-.47 to .98	.35	-1.09
HS	-0.64 to 0.91	.35	0.14	-.56 to .66	.30	-0.03
OPR	-0.83 to 0.94	.37	-0.83	-.72 to .96	.34	-0.96

Note. Due to missing data, the sample size in each of the 100 samplings was not always 100. The mean N was 74.56, SD = 6.20. The mean Λ in the 100 samplings was .36, SD = .07. The values within parentheses below the ranges for the first solution are the interquartile ranges.

Table 6

Standard Deviation and Other Statistics Relative to Standardized and Structure Coefficients Concerning Variable Set 2 for DA Runs

	β			r_s		
Variable	Range	SD	Skewness	Range	SD	Skewness
OPR	0.92 to 1.05	.02	-0.23	.92 to 1.00	.01	-2.54
	(0.98 to 1.00)			(.99 to 1.00)		
OB	-0.34 to 0.40	.14	-0.24	-.04 to .41	.10	0.06
	(-0.04 to 0.16)			(.11 to .25)		

Note. Due to missing data, the sample size in each of the 100 samplings was not always 150. The mean N was 91.36, SD = 5.66. The mean Λ in the samplings was .50, SD = .06. The values within parentheses below the ranges are the interquartile ranges.

Table 7

Standard Deviation and Other Statistics Relative to Standardized and Structure Coefficients Concerning Variable Set 1 for CA Runs

		β			r_s	
Variable	Range	SD	Skewness	Range	SD	Skewness
			Solution 1			
HA	-1.01 to 1.02	.66	-0.34	-.96 to 1.00	.67	-0.39
	(-0.36 to 0.86)			(-.34 to .88)		
NC	-0.86 to 0.86	.45	0.22	-.95 to .91	.53	0.20
	(-0.56 to 0.17)			(-.65 to .29)		
NS	-0.96 to 1.02	.49	0.31	-.98 to .95	.57	0.21
	(-0.51 to 0.26)			(-.62 to .36)		
L	-1.08 to 1.10	.65	-0.36	-.99 to 1.00	.70	-0.28
	(-0.42 to 0.85)			(-.54 to .89)		
E	-1.08 to 1.08	.71	-0.27	-1.00 to 1.00	.75	-0.15
	(-0.46 to 0.83)			(-.64 to .87)		
			Solution 2			
HA	-0.88 to 1.05	.54	-0.86	-.87 to .99	.52	-0.73
NC	-0.93 to 1.00	.51	0.30	-.88 to .95	.52	0.29
NS	-1.07 to 1.05	.55	0.16	-.98 to 1.00	.56	0.24
L	-1.11 to 1.08	.64	-1.11	-.96 to 1.00	.56	-0.97
E	-1.09 to 1.11	.73	-0.32	-1.00 to 1.00	.66	-0.13

Note. HA, NC, and NS constituted one variable set, and L and E constituted the second variable set. Due to missing data, the sample size in each of the 100 samplings was not always 100. The mean N was 63.15, SD = 4.02. The mean Λ in the 100 samplings was .67, SD = .09.
The values within parentheses below the ranges for the first solution are the interquartile ranges.

Table 8

Standard Deviation and Other Statistics Relative to Standardized and Structure Coefficients Concerning Variable Set 2 for CA Runs

Variable	β Range	β SD	β Skewness	r_s Range	r_s SD	r_s Skewness
			Solution 1			
OCCA	-0.96 to 1.02	.46	0.70	-.99 to 1.00	.54	0.70
	(-0.47 to 0.07)			(-.65 to .21)		
NS	-0.41 to 0.61	.21	0.38	-.68 to .72	.38	0.38
	(-0.16 to 0.17)			(-.39 to .26)		
M	-1.01 to 1.08	.77	-0.55	-1.00 to .99	.84	-0.54
	(-0.80 to 0.83)			(-.81 to .94)		
OPR	-0.99 to 0.97	.46	-0.64	-1.00 to .99	.53	-0.73
	(-0.13 to 0.47)			(-.11 to .62)		
PA	-1.01 to 1.04	.82	-0.54	-1.00 to 1.00	.86	-0.58
	(-0.86 to 0.92)			(-.85 to .97)		
			Solution 2			
OCCA	-1.08 to 1.14	.65	-1.83	-.98 to 1.00	.57	-1.57
NS	-0.45 to 0.43	.20	0.36	-.58 to .52	.24	0.13
M	-1.02 to 1.09	.52	-0.82	-.81 to .98	.42	-0.41
OPR	-1.19 to 1.04	.66	1.79	-1.00 to .98	.60	1.63
PA	-0.89 to 1.14	.52	-0.57	-.75 to .96	.43	-0.28

Note. OCCA, NS, and M constituted one variable set, and OPR and PA constituted the second variable set. Due to missing data, the sample size in each of the 100 samplings was not always 100. The mean N was 59.63, SD = 4.23. The mean Λ in the 100 samplings was .35, SD = .08. The values within parentheses below the ranges for the first solution are the interquartile ranges.

Discussion

No hypothesis was clearly supported by the results. For the DA runs, the stability of β was most often found to be less than the stability of the corresponding r_s. However, the reverse was true for the CA runs. The results for the CA runs somewhat contradict the results from the first writer's previous studies and what the literature usually suggests.

For the DA runs, the stabilities of the standardized and structure coefficients for the first solution were most often greater than the corresponding stabilities for the second solution. However, the reverse was generally true for the CA runs. The results for the CA runs somewhat contradict the results from the first writer's previous studies and what "common sense" would suggest.

The results for the least complex situation studied (DA with two discriminating variables) suggest greater stability of β and r_s than for more complex situations. Further study of this observation is warranted.

With the exception of the results from the second set of DA runs, the results -- as was true with the first writer's previous studies -- suggest alarmingly low stabilities for both β and r_s. This should not suggest that the stabilities of β and r_s would typically be greater in DA than in CA although this generality may be true. A plausible explanation regarding the better performance of β and r_s in the second set of DA runs is that of all the four sets studied this second DA set contained the lowest number of variables. That for this set the sample sizes were larger is thought by us to be of small effect.

The continuing evidence gathered regarding the generally low stability of β adds to its unattractiveness in CA and DA. However, the writers rebuke some of the previous criticisms of β in that (a) it has interpretive value evidenced, in part, by its somewhat frequent use, (b) its interpretation is different from the interpretation of r_s, and (c) it does not appear to be even close to universally less stable than r_s. Furthermore, in many situations β provides more useful information than does r_s.

Standardized coefficients already have a "bad" reputation. The writers, however, suggest not going so far as to avoid their use but to utilize caution in their interpretation. Furthermore, if use of β is to be avoided similar concern would also apply to r_s. Additional study of the difference in stability of the coefficients is warranted -- especially study of their apparently often alarmingly low performance. Some alternatives to their standard use exist -- jackknife and bootstrap procedures, for example -- but the alternatives have their own sets of limitations.

The writers have no explanation at this point concerning the results that pertain to the difference between the Solution 1 and Solution 2 stabilities of both statistics. When poorer stability performance of both statistics relative to Root 2 was found in the first writer's previous studies, the writer explained that the results were expected because roots beyond the first root contain more "error" as reflected by the declining R_c^2 with each subsequent root. The first writer's numerous observations across several years suggest that this generality is most true when the number of roots is low. More study of this issue is also warranted.

The somewhat unexpected results may be in part due to cases excluded from analysis due to missing data. The loss of the cases was assumed to be random -- which is consistent with most multivariate data-analysis practices. However, the skewness and other statistics for the cases actually utilized in each CA and DA run likely differed from the statistics reported for all the original cases. More attention to this issue in future studies is warranted.

The writers also considered that the unexpected results may be attributed to unique samples that resulted from original variables whose SDs were "too low." This prompted the writers' inspection of the SDs of the original variables -- which was lowest for L, SD = 0.57. The writers concluded that the SDs for all the original variables were sufficient.

While the search to find the best method to study the stability of the statistics that pertain to this study is likely to be accompanied by frustration, researchers must have an open mind to the advantages and disadvantages of the several approaches -- in terms of their own work as well as their critiques of the works of others. The writers are still not satisfied with utilizing SDs as the primary criterion but have as yet found no clearly more attractive alternative.

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